MATH 208 C — MIDTERM 2 — Autumn 2022

NAME: _____

To make it possible for Gradescope to recognize you, please write your name:

(1) **clearly** in **CAPITAL LETTERS**,
(2) **exactly on the line above**,
(3) use the **name you are registered under** for this class.

(1) Please put away all phones and earphones in your bag.
(2) There are 4 problems.
(3) Show all of your work and justify your answers.
(4) Write clearly.

(1) (a) Let C be the unit square in $\mathbb{R}^2$ with corners $(0,0),(1,0),(0,1),(1,1)$. What is the image of C under the linear transformation $T(x) = Ax$ where

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 0 & 0 \end{bmatrix}?$$

Draw a picture of the image and mark everything clearly.

(b) Write down the linear transformation that will take C and **first** apply T to it and **then reflect** the image of C under T across the $(\mathbf{x_2}, \mathbf{x_3})$-plane.
The transformation should be written in full with domain, codomain and matrix. The matrices in the composition must be written out but you don't need to multiply them out.

(2) (a) Is the following a linear transformation? Give reasons for your answer.

$$\begin{array}{rccc} T: & \mathbb{R}^2 & \longrightarrow & \mathbb{R}^2 \\ & \begin{pmatrix} x \\ y \end{pmatrix} & \mapsto & \begin{pmatrix} 2x+y \\ x+1 \end{pmatrix} \end{array}$$

(b) Is the following a vector space? Give reasons for your answer.

$$S = \left\{ \begin{pmatrix} x \\ y \end{pmatrix} \,:\, y = x^2 \right\}.$$

(3) Let T be the linear transformation such that $T(x) = Ax$ where A and its echelon form B are shown below.

$$A = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 0 & 4 \\ -1 & 6 & -8 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}.$$

(a) Compute a basis for the kernel of T.

(b) Compute a basis for the range of T.

(c) Is T invertible? If yes, compute T^{-1}. If not, say why not.

(4) In the following questions, all transformations must be written fully with domain, codomain and matrix. Let

$$S = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} : \begin{array}{r} x + 2y + 3z - w = 0 \\ -z + w = 0 \end{array} \right\}.$$

(a) What is the dimension of S?

(b) Find a linear transformation P such that $S = kernel(P)$.

(c) Find a linear transformation Q such that $S = range(Q)$.